One GeV Acceleration with Laser Wake-Field in the Linear Regime ⋆

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Abstract

We derive an expression for the maximum energy gain of an accelerated electron, in the limit that the plasma wave created by a laser wake is linear both along the longitudinal direction and in the transverse plane, and with a maximum laser power lower than the critical power for relativistic self-focusing. With an available power of 300 TW, the energy gain is of 1 GeV.

1 Introduction

Laser particle acceleration is actively being studied at the present time, in the hope that the high electric field will permit the design of ultra high energy linear accelerators of reasonable length.

Several schemes make use of a plasma as a transformer of the transverse electromagnetic wave into a longitudinal electronic plasma wave (EPW), with a phase velocity close to the velocity of light. A bunch of charged particles can then be accelerated by the longitudinal electric field of the EPW.

The laser wake-field (LWF) [1] is a particularly efficient and elegant approach in which a single short laser pulse excites a high amplitude EPW. EPW with electric fields up to 10 GV/m and amplitude of $\delta \approx 0.5$ have been produced recently and diagnosed by two pulse frequency-domain interferometry [2].

We present here an attempt to compute the maximum possible electron energy gain at a given available laser power.

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While the laser pulse is passing by a plasma electron, the electron undergoes a forward force as long as the laser intensity is increasing, and a backward force afterwards. The final relative perturbation δ of the electron density n of the plasma is maximum when the plasma wave frequency ω_p is tuned to the time separation between the two pushes, that is when $\omega_p\tau_0 = 2$, where τ is the FWHM pulse length of the laser and $\tau_0 = 4\sqrt{\ln 2}\tau$.

In contrast with the beat-wave excitation of the plasma wave, the resonance condition is rather a loose one, as the maximum electric field of the EPW goes as $(\omega_p\tau_0)^2 \exp\left(-(\omega_p\tau_0)^2/4\right)$. In the following article, we will set the electron density at its optimal value.

LWF acceleration has been demonstrated recently [3]. The signal was identified by its dependence on the electron density of the plasma.

2 Linear and non-linear "longitudinal" EPW

In the linear regime, i.e. at low amplitude $\delta < 1$, the plasma wave has a sinuoidal shape, and the maximal electric field and the amplitude of the plasma wave vary linearly with the laser energy. The value of the field in the linear approximation is limited to $E_z = E_0$ for $\delta = 1$, where $E_0 = mc\omega_p/e$ is the wave-breaking limit with the 1D cold plasma approximation.

At higher laser energy, the wave is distorted : the electric field has a "saw-tooth" profile with a slow rise and a steep drop. On the steepest part of the phase segment, there exist a very high electric field, that now depends on the square of the laser energy. However, in reality, it is unlikely that these non-linear EPW will be useful for particle acceleration. Not only do their wave-length and phase velocity vary with intensity, and therefore with space and time, but 2D simulations of a hot plasma have shown that under these conditions wave-breaking still takes place for values of the field close to E_0 (a discussion and exhaustive references can be found in ref.[4]).

Therefore we take $\delta = 1$ as the limit of the linear regime.

3 Transverse EPW

The perturbation of the electronic density of the plasma also has a transverse component $\delta_\perp$. When the computed value of $\delta_\perp$ greatly exceeds unity, the electrons are completely expelled from the high intensity region, and the longitudinal plasma wave cannot take place : the longitudinal electric field is then very weak [5].

The ratio $\delta_{\perp}/\delta$ is proportionnal to w_0/λ_p, where λ_p is the EPW wave-length and w_0 is the size of the waist.

For an efficient acceleration of relativistic particles, one needs an EPW with a high phase velocity, i.e. a high Lorentz factor $\gamma_p = \lambda_p/\lambda$, where λ is the laser wave-length. This leads to EPW with large wave-lengths. On the other hand, strong focusing is needed so that the acceleration length will be shorter than the dephasing length of the injected particles *wrt* the EPW.

Therefore the value of $\delta_{\perp}/\delta$ can often be larger than unity in actual acceleration experiments. We therefore have to specify explicitly that $\delta_{\perp}$ be lower than one.

4 Electron energy gain

The maximum energy gain of an electron accelerated in the EPW is given by

$$\Delta W = e \int E_z \, \mathrm{d}z \tag{1}$$

where we have assumed that the electron stays in phase with the wave, that is close to the maximum of the field throughout the plasma. For a laser beam with a gaussian transverse profile, this results in $\Delta W = \pi e z_R E_{z,max}$, where z_R is the Rayleigh length of the laser. From this expression, and with a plasma frequency tuned to the pulse length, we get

$$\Delta W = P \frac{1}{M^2} \frac{\lambda}{\lambda_p} \frac{r_e}{c} \frac{4\pi^{3/2}}{\mathrm{e}} \tag{2}$$

that is

$$\Delta W[\mathrm{MeV}] = 481 \frac{1}{M^2} \frac{\lambda}{\lambda_p} P[\mathrm{TW}] \tag{3}$$

Here, e is exp (1), M^2 is the Siegman factor of the laser beam, P is the available laser power at maximum, and r_e is the classical radius of the electron.

This leads obviously to the use of shorter pulses, that is to a denser plasma, and to lower values of γ_p. As an example, for 35 fs laser pulses at $\lambda = 0.8\mu$m, $\Delta W = 1$GeV is readily obtained for a laser energy $E = 2$J.

5 Beam blow up

Actually in the given example, the maximum power is $P = 53.7$ TW, while the critical power for self-focusing is

$$P_{\rm crit} = 2\,\frac{mc^3}{r_e}\left(\frac{\omega}{\omega_p}\right)^2 \tag{4}$$

that is $P_{\rm crit}[\mathrm{GW}] = 17.4\ \gamma_p^2$. For $\tau = 35$fs, we have $\gamma_p = 24.8$, and $P_{\rm crit} = 10.4$TW. The plasma will "blow up" the laser beam. Self-focusing and the associated instabilities therefore strongly limit the effectiveness of high gradient LWF acceleration, as was already noted in [6].

6 The configuration

We then search an optimum of ΔW under the constraint $P = P_{\rm crit}$, that is :

$$2\,\frac{E}{\tau}\sqrt{\frac{\ln 2}{\pi}} = 2\,\frac{mc^3}{r_e}\left(\frac{\omega}{\omega_p}\right)^2 \tag{5}$$

where E is the energy of the laser pulse. We obtain

$$E = \frac{\tau^3 mc^5}{\lambda^2 r_e}\frac{\pi^{5/2}}{4(\ln 2)^{3/2}} \tag{6}$$

so :

$$\Delta W = \frac{\tau}{\lambda}mc^3\frac{4\pi^{5/2}}{\mathrm{e}\,M^2\sqrt{\ln 2}} \tag{7}$$

We want now to *increase* τ, and therefore E. At $\delta = 1$, the intensity I of the laser is fixed and w_0 and z_0 increase with τ. Finally, the limit is given by the available power P.

The configuration is therefore determined by :

- $\delta = 1$
- $P = P_{\rm crit}$
- $\lambda = 0.8\mu$m

and by the value of P. The obtained expressions of the parameters are listed in column 1 of table 1. Column 2 lists the corresponding expressions in usual units, while the numerical values for $P = 300$ TW are presented in column 3. The derived values of the parameter seem to be sensible, an eventual ex-

Table 1
Values of the parameters of an experiment with $\delta = 1$ and $P = P_{\rm crit}$.

$\tau = \frac{\lambda}{\pi c}\left[2\ln 2\frac{P r_e}{mc^3}\right]^{\frac{1}{2}}$	$\tau[\mathrm{fs}] = 13.4\ \lambda[\mu\mathrm{m}]\,(P[\mathrm{TW}])^{\frac{1}{2}}$	185 fs
$E = \frac{\lambda}{c}\left[\frac{r_e}{2\pi mc^3}\right]^{\frac{1}{2}} P^{\frac{3}{2}}$	$E[\mathrm{J}] = 0.0142\ \lambda[\mu\mathrm{m}]\,(P[\mathrm{TW}])^{\frac{3}{2}}$	59.2 J
$\Delta W = \frac{8\pi^2}{\mathrm{e}\,M^2}\left[\frac{r_e mcP}{2\pi}\right]^{\frac{1}{2}}$	$\Delta W[\mathrm{MeV}] = 63.5\,(P[\mathrm{TW}])^{\frac{1}{2}}$	1100 MeV
$I_{max} = \mathrm{e}\sqrt{\pi}\frac{mc^3}{\lambda^2 r_e}$	$I_{max}[\mathrm{Wcm}^{-2}] = 4.19\ 10^{18}/(\lambda[\mu\mathrm{m}])^2$	$6.55\ 10^{18}\mathrm{Wcm}^{-2}$
$\gamma_p = \left[\frac{P r_e}{2mc^3}\right]^{\frac{1}{2}}$	$\gamma_p = 7.58\,(P[\mathrm{TW}])^{\frac{1}{2}}$	131.
$\lambda_p = \lambda\left[\frac{P r_e}{2mc^3}\right]^{\frac{1}{2}}$	$\lambda_p = 7.58\ \lambda[\mu\mathrm{m}]\,(P[\mathrm{TW}])^{\frac{1}{2}}$	105 μm
$w_0 = \lambda\left[\frac{2}{\pi^{\frac{3}{2}}\,\mathrm{e}}\frac{P r_e}{mc^3}\right]^{\frac{1}{2}}$	$w_0[\mu\mathrm{m}] = 3.90\ \lambda[\mu\mathrm{m}]\,(P[\mathrm{TW}])^{\frac{1}{2}}$	54 μm
$z_0 = \frac{2}{\mathrm{e}\sqrt{\pi}M^2}\frac{r_e}{mc^3}P\lambda$	$z_0[\mu\mathrm{m}] = 47.7\ \lambda[\mu\mathrm{m}]P[\mathrm{TW}]$	11.5 mm
$(E_z)_{max} = \sqrt{2\pi}\frac{mc^2}{r_e}\frac{1}{\lambda\sqrt{P}}\left[\frac{c}{\epsilon_0}\right]^{\frac{1}{2}}$	$(E_z)_{max}[\mathrm{V/m}] = \frac{4.24\ 10^{11}}{\lambda[\mu\mathrm{m}]\,(P[\mathrm{TW}])^{\frac{1}{2}}}$	30.6 GV/m
$n = \frac{2\pi mc^3}{r_e^2\lambda^2 P}$	$n[\mathrm{cm}^{-3}] = \frac{1.94\ 10^{19}}{(\lambda[\mu\mathrm{m}])^2\,P[\mathrm{TW}]}$	$1.01\ 10^{17}\mathrm{cm}^{-3}$
$o = M^2\left[\frac{\mathrm{e}}{2\sqrt{\pi}}\frac{mc^3}{P r_e}\right]^{\frac{1}{2}}$	$o = M^2\frac{0.0817}{(P[\mathrm{TW}])^{\frac{1}{2}}}$	0.00472
$\frac{\delta_\perp}{\delta} = \left(\frac{E_r}{E_z}\right)^2 = \frac{\mathrm{e}}{2\sqrt{\pi}}$		0.77

ception being the very small apperture o. Actually, at a maximum fluence of 400mJcm^{-2}, we have a beam radius before focusing of $w_i = 5.2$cm, and a focal length $f = 11$ m, which seems manageable.

Note that the fulfilment of two of the given constraints, $\delta = 1$ and $P = P_{\rm crit}$, imply the fulfilment of the third one $(E_r/E_z)_0 = 0.876$, that is $\delta_\perp < \delta = 1$.

The ultra-relativistic dephasing length is $L_\varphi = \gamma_p^3\lambda = 1.8$ m. The depletion length is $L_d = \tau c(E_{\rm laser}/E_z)^2$ where $E_{\rm laser}$ is the maximal electric field of the laser. For $\delta = 1$, this results in

$$L_d = \frac{(c\tau)^3}{\lambda^2}\frac{\mathrm{e}\,\pi^{3/2}}{2\ln 2} \approx 2.9\ \mathrm{m} \tag{8}$$

The dephasing length and the depletion length are of the same order of magnitude, as is expected for laser pulses at the limit of the relativistic regime.

7 Injection Energy

The injection energy of the electrons is chosen so that the linear dephasing factor $\kappa = \exp\left(-\pi L/(2L_c)\right)$ will be close to 1. We invert to get :

$$\gamma_0 = \left(\frac{1}{\gamma_p^2} - \frac{\lambda_p \ln\kappa}{\pi z_0}\right)^{-\frac{1}{2}} \tag{9}$$

that is $\gamma_0 = 52$ for $\kappa = 0.9$. This corresponds to an injection energy of 27 MeV.

8 Conclusion

- LWF acceleration can be optimised for a given available maximum power by
  - $\delta = 1$
  - $\delta_\perp < 1$
  - $P = P_{\text{crit}}$
- Indeed when the first and the last constraint are satisfied, the second one is also satisfied : $\delta_\perp = 0.77$
- With a maximum power of 300 TW and an electron injection energy of 27 MeV, the energy gain is 1 GeV.

If it is possible to channel the laser beam in a preformed plasma over a fraction of the depletion length – say 1 m – as suggested by J. Rosensweig during the workshop, the energy gain would be as high as 30 GeV (see also [7]).

References

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